Exercise 27

Evaluate the integral.

$$\int_0^1 (u+2)(u-3)\,du$$

Solution

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where F is an antiderivative of f.

$$\begin{aligned}
\int_0^1 (u+2)(u-3)\,du &= \int_0^1 (u^2 - u - 6)\,du \\
&= \int_0^1 u^2\,du - \int_0^1 u\,du - \int_0^1 6\,du \\
&= \left(\frac{u^3}{3}\right)\Big|_0^1 - \left(\frac{u^2}{2}\right)\Big|_0^1 - (6u)\Big|_0^1 \\
&= \frac{1}{3}\,(u^3)\Big|_0^1 - \frac{1}{2}\,(u^2)\Big|_0^1 - 6\,(u)\Big|_0^1 \\
&= \frac{1}{3}(1^3 - 0^3) - \frac{1}{2}(1^2 - 0^2) - 6(1-0) \\
&= \frac{1}{3} - \frac{1}{2} - 6 \\
&= -\frac{37}{6}
\end{aligned}$$